c) The displacement is a maximum when the term in parentheses in the denominator is zero; the denominator is the sum of two squares and is minimized when \( x = vt \), and the maximum displacement is \( A \). At \( x = 4.50 \text{ cm} \), the displacement is a maximum at \( t = (4.50 \times 10^{-2} \text{ m})/(20.0 \text{ m/s}) = 2.25 \times 10^{-3} \text{ s} \). The displacement will be half of the maximum when \( (x - vt)^2 = A^2 \), or \( t = (x \pm A)/v = 1.75 \times 10^{-3} \text{ s} \) and \( 2.75 \times 10^{-3} \text{ s} \).

d) Of the many ways to obtain the result, the method presented saves some algebra and minor calculus, relying on the chain rule for partial derivatives. Specifically, let \( u = u(x, t) = x - vt \), so that if

\[
\frac{\partial f}{\partial x} = \frac{dg}{du} \frac{\partial u}{\partial x} = \frac{dg}{du} \quad \text{and} \quad \frac{\partial f}{\partial t} = \frac{dg}{du} \frac{\partial u}{\partial t} = -\frac{dg}{du}v.
\]

(In this form it may be seen that any function of this form satisfies the wave equation; see Problem 15.59.) In this case, \( y(x, t) = A^3(\sqrt{A^2 + u^2})^{-1} \), and so

\[
\begin{align*}
\frac{\partial y}{\partial x} &= -2A^3u \frac{1}{(A^2 + u^2)^2}, \\
\frac{\partial^2 y}{\partial x^2} &= -2A^3(\sqrt{A^2 - 3u^2}) \frac{1}{(A^2 + u^2)^3}, \\
\frac{\partial y}{\partial t} &= v \frac{2A^3u}{(A^2 + u^2)^2}, \\
\frac{\partial^2 y}{\partial t^2} &= -v^2 \frac{2A^3(A^2 - 3u^2)}{(A^2 + u^2)^2},
\end{align*}
\]

and so the given form for \( y(x, t) \) is a solution to the wave equation with speed \( v \).