Zero-Dimensional Energy Balance Model

Motivation

The overall goal of this exercise is to make predictions about the global temperature of the Earth. We will achieve this by investigating the balance between how radiative energy is absorbed, reflected, transported, and stored by the Earth. Interesting phenomena, such as the greenhouse effect and ice-albedo feedback can be studied using such models. These models are classified as energy balance model and are similar to the hot/cold can experiment discussed earlier.

In the present exercise we consider only a model for the average global temperature in which we treat the entire Earth as a single point. Once we have familiarized ourselves with this model, we can later work with a one-dimensional model of the Earth which is broken up into 10° latitude regions. This will allow us to determine not just the global temperature, but the temperature in each of 9 latitude regions in the Northern Hemisphere. After that we can look at at two-dimensional model that will allow us to study the variation of temperature both in the latitudinal and longitudinal direction, over the entire surface of the Earth.

Theory - Steady-State Model

We consider the following model for the average global temperature in which we treat the entire Earth as a single point. In an energy balance model, the main goal is to account for all heat flows in ($P_{\text{gain}}$) and out ($P_{\text{loss}}$) of the system. If these are balanced ($P_{\text{gain}}=P_{\text{loss}}$), the system will be in a steady state and at a constant temperature. If the heat flows are not balanced, the temperature of the system will change.

The heat flow to the Earth arrives from the Sun. The Solar Constant ($S_o$) is the amount of energy arriving (during a 1 second period on a 1 square-meter area oriented perpendicular to the sun’s rays) at the upper atmosphere. The annual average value of the solar constant is $S_o=1370$ W/m². This energy arrives primarily in the form of visible light, with some smaller amounts of infrared and ultraviolet radiation.

A fraction of the sun’s radiation is immediately reflected back into space, either from the atmosphere, clouds, or the Earth’s surface. The Albedo ($\alpha$) of the Earth is the fraction of the Sun’s radiation which is reflected back into space. Thus, the net amount of solar radiation arriving on a 1 m² area (perpendicular to sun) on the earth’s surface is:

$$S_o(1- \alpha).$$
From the point of view of the Sun, the Earth appears to be a disk with a radius $R_E$, so the total amount of power absorbed by the whole Earth is the product of the arriving solar radiation times the area of a disk the size of the Earth:

$$P_{gain} = \pi \; R_E^2 S_0 \; (1-\alpha).$$

That describes the gain of energy, but what about the loss of energy? Any object at a temperature $T_K$ (in Kelvin) will emit thermal radiation at a rate given by the Stefan-Boltzmann Law:

$$P_{loss} = \varepsilon \; \sigma \; T_K^4$$

times its surface area. The factor $\varepsilon$ is the emissivity (approximately 1), $\sigma$ is the Stefan-Boltzmann constant, and the total surface area of the spherical Earth is $(4 \pi R_e^2)$. Recall that a temperature in Kelvin is $T_K = T_0 + T_C$ where $T_C$ is the temperature in Centigrade and $T_0 = 273.15$.

In the steady state, the incoming radiation must balance the outgoing radiation. This leads to an energy balance equation for $P_{gain} = P_{loss}$:

$$\pi \; R_e^2 \; S_0 \; (1-\alpha) = (4 \pi \; R_e^2) \; \varepsilon \; \sigma \; (T_C + T_0)^4.$$

Solving for $T_C$ gives the following equation:

(Eqn. 1): $T_C = [S_0 \; (1-\alpha) / (4 \; \varepsilon \; \sigma)]^{1/4} - T_0$.

Where the symbols are defined as:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_C$</td>
<td>Earth's Surface Temperature (in Centigrade)</td>
</tr>
<tr>
<td>$S_0$</td>
<td>Solar Constant (1370 W/m$^2$)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Albedo (fraction of incident solar radiation reflected (about 0.32))</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzmann Constant (5.67E-8 W/(m$^2$K$^4$))</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Conversion Constant (from Kelvin to Centigrade (273.15))</td>
</tr>
</tbody>
</table>

**Numerical Model**

To study this, Tom Huber has developed a Matlab script to calculate $T_C$. To illustrate some of the Graphical User Interface (GUI) capabilities of Matlab, you may use the script `ebm_0_dim_gui.m` (along with `ebm_o_dim.m`) which allow you to easily change some of the parameters. To run this script in Matlab, type `ebm_0_dim_gui` and a GUI will appear (provided you have properly set your Matlab path). Enter an initial guess for the temperature in the upper left. Make sure that the 'Blackbody' button in the lower left
is on (we want to compute pure 'Blackbody' temperatures). Click the 'Calculate' button on the right. (If you do not see all these buttons then you must manually increase the size of the GUI window until they appear).

Experiment 1 - Blackbody Earth

Using the model calculate \( T_C \) for the Earth. Is this a reasonable value, or is this model Earth temperature colder than the real average Earth temperature (i.e., about 15 degrees Centigrade)?

Activate the 'Advanced Controls' and turn off the 'Blackbody' control. The Earth then becomes a 'Greybody', or in other words, a non-perfect emitter of longwave radiation. Recompute the temperature \( T_C \) of the Earth as a greybody. Is this a more reasonable value? How do you explain the difference between these two model-derived temperatures?

Experiment 2 - Other Blackbody Objects

The intensity of solar radiation received at a given solar system object is proportional to \( 1/r^2 \), where \( r \) is the mean distance between the Sun and the object. This inverse-square law reflects the fact that you get further away from the Sun then the fixed amount of energy emitted at the surface of the Sun is spread out over an ever-larger surface area (i.e., an area proportional to the quantity \( r^2 \)) in inter-planetary space. It follows then that the solar radiation received at any other object, \( S_o(\text{object}) \) in the solar system can be calculated since we know the radiation received at the Earth, \( S_o \):

(Eqn. 2): \( S_o(\text{object}) = S_o \frac{r_{\text{Earth}}^2}{r_{\text{planet}}^2} \)

where \( r_{\text{Earth}} \) is the mean Earth-Sun distance (here taken to be 1 AU (Astronomical Unit)) and \( r_{\text{planet}} \) the mean planet-Sun distance. For instance, since Mars is about 1.5 times further from the sun (i.e, it is found at a distance of 1.5 AU), it will receive roughly \( 1.0/(1.5)^2 = 0.444 \) times the solar radiation received on Earth. In these estimations we ignore differences in the size of the planets on the total amount of collected solar energy.

For each of the solar system objects listed in the table below, compute the solar constant \( S_o(\text{planet}) \) using the mathematical relation given as Eqn. 2. Report the appropriate solar constant in a copy of the table below having a new column called 'Solar Constant'.

Next compute the respective blackbody temperature for each solar system object listed in the table below using the model. Ensure that the 'Blackbody' control is on. Once you have collected the model output data of steady-state temperature for each solar system object listed, then enter the results into a copy of the table below having a new column called 'Blackbody Temperature'. You should merge this table with the one you created above so that only one new table is created.
Plot the results in a graphical manner such that the 'x-axis' represents the distance of the solar system object from the Sun and the 'y-axis' represents to the mean blackbody steady-state temperature of the object. What trend is apparent? Overlay on that plot (using a different color marker or line), the actual solar system object temperatures as taken from the table below. For which planet is the difference the greatest? What might account for the differences between the modeled temperature and the observed?

<table>
<thead>
<tr>
<th>Solar System Object</th>
<th>Distance (AU)</th>
<th>Albedo</th>
<th>Ave. Temperature</th>
<th>Atmosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>Venus</td>
<td>0.723</td>
<td>0.76</td>
<td>425 °C</td>
<td>95 Atm, 96% CO₂</td>
</tr>
<tr>
<td>Earth</td>
<td>1.000</td>
<td>0.32</td>
<td>15 °C</td>
<td>1 Atm, N₂, O₂, Trace H₂O, CO₂</td>
</tr>
<tr>
<td>Mars</td>
<td>1.524</td>
<td>0.16</td>
<td>-50 °C</td>
<td>0.02 Atm, 95% CO₂</td>
</tr>
<tr>
<td>Europa (A Moon of Jupiter)</td>
<td>5.203</td>
<td>0.64</td>
<td>-145 °C</td>
<td>Essentially None</td>
</tr>
</tbody>
</table>

For additional data on these and other objects in solar system see JPL.

**Theory - Greenhouse Effect**

The steady-state situation given by Eqn. 1 is not fully correct because it leads to a temperature of about -16 °C for the Earth which is well below the mean global temperature of about 15°C. The major reason for this difference is the *Greenhouse Effect* in the atmosphere. The calculation for the temperature of Mars, with very little atmosphere, is in very good agreement with the observed temperature. Venus, on the other hand, with its very thick atmosphere of CO₂ has a large greenhouse effect leading to a much higher temperature than predicted by Eqn. 1. Incidentally, while the surface temperature of Europa is quite cold there exists, nonetheless, gravitational tidal forces cause the interior to be warm enough that it appears that there may be liquid water beneath its ice crust. We do not consider 'geothermal' energy sources in the present exercise but you should be aware that they can, in certain circumstances, make substantial contributions to the overall energy balance of a solar system object.

The greenhouse effect is due to absorption and re-emission of infrared radiation by the Earth's atmosphere. As mentioned earlier, the Earth receives visible light from the sun and emits heat in the form of infrared radiation. The Earth's atmosphere is nearly transparent in the visible region of the spectrum, but gases (such as water vapor, CO₂ and others) in the atmosphere can absorb infrared radiation. When a gas molecule (such as CO₂) absorbs some infrared radiation, it ends up in an excited state and eventually it decays to the ground state. In the process, it will re-emit energy in the infrared in a random direction (some of it back towards the earth). The net result is that some of the infrared radiation is "reflected" back towards the earth, thus effectively reducing the loss of heat from the earth.

This is called the greenhouse effect because a similar mechanism occurs in a
greenhouse (or a closed car in a parking lot on a sunny day). Visible light passes into the greenhouse and is absorbed by material inside. This material re-emits energy in the infrared, which is reflected back by the glass of the greenhouse (or car). Because of this trapping of heat, it is possible for a sealed greenhouse (or car) to reach a temperature far higher than the outside temperature.

The major contributor to the greenhouse effect in the atmosphere is water vapor (because it is the most abundant contributor). The next leading contributor is CO₂, with others including ozone, N₂O, methane and CFC’s. Human activities are causing changes in the concentrations of these gases, which may lead to increased greenhouse effects.

Since all of our Earth temperatures are within about +/- 20°C, it is possible to re-write the Stefan-Boltzmann Law describing the Earth's emitted longwave radiation using the binomial expansion: (1+x)ⁿ which is approximately equal to (1+nx) if x is much less than 1. Therefore, we can write:

\[ T_K^4 = (T_o + T_C)^4 = T_o^4 (1 + T_C / T_o)^4 \]

which is approximately

\[ T_o^4 (1 + 4 T_C / T_o). \]

This allows us to re-write the Stefan-Boltzmann Law as:

\[ P_{\text{loss}} = (4 \ \pi \ R_E^2) (A + B^* T_C) \]

where the two constants are:

\[ A = \varepsilon \ \sigma \ T_o^4 = 315 \ \text{W m}^{-2} \]

for a blackbody, and

\[ B = 4 \ \varepsilon \ \sigma \ T_o^3 = 4.6 \ \text{W m}^{-2} \ ^{\circ} \text{C}^{-1} \]

again, for a blackbody. The greenhouse effect can be included by modifying the values used for A and B. Smaller values imply a stronger greenhouse effect.

As stated earlier, in the steady state incoming radiation must balance the outgoing radiation. The accommodation of the greenhouse effect leads to a modified steady-state energy balance equation for \( P_{\text{gain}} = P_{\text{loss}} \):

\[ \pi \ R_E^2 S_o (1 - \alpha ) = (4 \ \pi \ R_e^2) (A + B^* T_C). \]

Solving for \( T_C \) gives the following equation:
(Eqn. 3): \[ T = \frac{S(1 - \alpha)}{4} - A / B \]

where the symbols are defined as:

- \( T_C \): Temperature of the Earth (in Centigrade)
- \( S_o \): Solar Constant (1370 W/m\(^2\))
- \( \alpha \): Albedo (Fraction of incident solar radiation reflected (about 0.32))
- \( A \): Constant Coefficient for Longwave Radiation (A=204 W m\(^{-2}\))
- \( B \): Temperature-Dependent Coefficient for Longwave Radiation (B=2.17 W m\(^{-2}\) °C\(^{-1}\))

**Experiment 3 - Greenhouse Effect**

Calculate \( T_C \) (in Centigrade) with the addition of the greenhouse effect. Note that you have to put the 'Blackbody' control in the off position in order to have the 'A' and 'B' parameters have impact on the model. How much temperature change is due to the greenhouse effect (i.e., a blackbody solution versus a greybody solution)?

Some previous models use \( A = 202 \text{ W m}^{-2} \) and \( B = 1.45 \text{ W m}^{-2} \text{ °C}^{-1} \) (Budyko, 1969) or \( A = 212 \text{ W m}^{-2} \) and \( B = 1.6 \text{ W m}^{2} \text{ °C}^{-1} \) (Cess, 1976). How sensitive is the Earth's global temperature to these alternative values of the \( A \) & \( B \) parameters?

**Experiment 4 - Faint Young Sun Paradox**

It is widely believed that early in the Earth's history the solar constant was about 85% of its present value. Using a greybody earth with the standard model parameters for \( A \) & \( B \) what temperature do you obtain for the Earth under such a reduced solar intensity? It is believed that the Earth was as warm, or warmer, in its distant past than it is presently (this is generally referred to as the Faint Young Sun Paradox). Can you find a pair of \( A \) & \( B \) values such that the Earth's temperature rises to its present global average? For the values of \( A \) & \( B \) you have chosen does this imply a stronger greenhouse effect in Earth's distant past than at present or a weaker one?

**Theory - Transient-State**

The model described by Eqn. 3 is the steady-state (equilibrium) temperature of the Earth. Our experience indicates that the Earth is never in equilibrium during the short term (e.g., the local weather changes on the scale of hours). However, if all of the factors in Eqn. 3 remained the same, the yearly average temperature would be the same.

In reality, due to various natural and man-made effects, the factors in Eqn. 3 can have long-term changes. A volcanic eruption or large fires can modify the Earth's albedo. Human activity has changed the amount of greenhouse gases in the atmosphere, thus changing the \( A \) & \( B \) factors. We can re-run the model to find the new equilibrium temperature after changing some or all of these factors.
In some cases, the system will not be in thermal equilibrium, or in other words, the energy loss will not be equal to the energy gain. This will lead to a changing temperature in the system. We can modify our model to allow energy to be stored or released, thus making a transient-state (non-equilibrium) model.

A large fraction of the thermal energy storage which effects the climate is due to the upper layer of mixed water in the ocean (about the upper 70 meters). We can write the effective heat capacity $C_E$ of a 1 square-meter area of the earth as:

$$C_E = f \rho \ c \ h$$

where:

$f$ : Fraction of Earth Covered by Water (0.7)
$\rho$ : Density of Sea Water (1025 kg/m$^3$)
$c$ : Specific Heat of Water (4186 J/kg$^0$C)
$h$ : Depth of Ocean Mixed Layer (70 m)

Putting these together gives a heat capacity of $C_E = 2.08 \times 10^8$ J/m$^2$ $^0$C. So, to change the temperature of 1m$^2$ of the Earth by 1 $^0$C will take on the average $2.08 \times 10^8$ J of energy. This will tend to moderate the effects of "rapid" changes in any of the parameters governing our global equilibrium temperature. Thus it may take many years for the temperature to converge to a new equilibrium.

As discussed in the previous model of the hot/cold cans, the temperature change which results from a difference in $P_{gain}$ and $P_{loss}$ is given by:

(Eqn. 4):  $\delta T = (P_{gain} - P_{loss}) \frac{\delta t}{C_E}$

where:

$P_{gain} = S(1 - \alpha)/4$

$P_{loss} = A + B* T_C$

The various constants are defined as:

$T_C$  Temperature of Earth (in Centigrade)
$\delta t$  Time interval (as measured in seconds for each iteration)
$\delta T$  Temperature change during time interval $\delta t$
$S_o$  Solar Constant (1370 W/m$^2$)
$\alpha$  Albedo (Fraction of incident solar radiation reflected (about 0.32))
$A$  Constant Coefficient Longwave Radiation ($A=204$ W m$^{-2}$)
$B$  Temperature-Dependent Coefficient Longwave Radiation ($B=2.17$ W m$^{-2}$ $^0$C$^{-1}$)
Experiment 5 - Transient-State Temperature

Modify the given value for the Earth's heat capacity by making it a factor of 10 larger and then 10 smaller. Run the model and estimate the time in years for an equilibrium temperature to be reached. Reset the heat capacity to its original value and report the time in years for equilibrium to be established. In each of the above experiment ensure that the 'Blackbody' control is off and set the 'Initial Temp' control to 10.0 degrees Centigrade (for each model run). What is the steady-state temperature in each case. Explain the qualitative differences in each of the three model scenarios.

Experiment 6 - Volcanic Eruption

The sudden explosion of a volcano could release aerosols that might block out the incoming solar radiation. We can model this using the 'Lowered Solar Constant' control (under the 'Advanced Controls'). To perform this experiment you first need to set the 'Initial Temp' of the Earth to its steady-state value (approximately 15 degrees Centigrade). Consider a 5% decrease in the solar constant during a 5 year period after which it returns to its initial value. In this particular experiment 'First Year' is to be assigned as year 5 and 'Last Year' as year 10. What happens to the temperature? How about if this lasts for 10 years? What if there is a 10% decrease for 5 years? What about a 20% decrease for 1 year? Explain the results you obtain.

Theory - Temperature-Dependent Albedo

A refinement we introduce into our model is to put a temperature dependence on the albedo. The justification for this is that as the temperature of the Earth decreases, more of the water on the planet will freeze, which will increase its albedo (since the albedo of ice is about 0.6 compared to about 0.32 for land). We account for this in the following way:

\[
\alpha = \begin{cases} 
\alpha_{\text{Ice}} & \text{if } T_C < T_{\text{Ice}} \\
\alpha_{\text{Land}} & \text{if } T_C > T_{\text{Land}} \\
\alpha_{\text{Ice}} + (\alpha_{\text{Land}} - \alpha_{\text{Ice}}) \frac{(T_C - T_{\text{Ice}})}{(T_{\text{Land}} - T_{\text{Ice}})} & \text{otherwise}
\end{cases}
\]

where the constants are defined:

- \(\alpha_{\text{Ice}}\): Albedo of Ice (0.6)
- \(T_{\text{Ice}}\): Global Temperature at which entire Earth freezes (-10 C)
- \(\alpha_{\text{Land}}\): Albedo of Land (0.32)
- \(T_{\text{Land}}\): Global Temperature at which entire Earth melts to current state with small polar ice caps (+10 C)

Experiment 7 - Temperature-Dependent Albedo
To perform this experiment you will need to activate the 'Temperature Dependent' albedo control. What is the final temperature if the initial temperature is 13 degrees? Decrease the initial temperature from 13 degrees to 3 degrees in 1 degree decrements. Report the equilibrium temperature for each initial temperature in a graphical format. Repeat the experiment but with the 'Temperature Dependent' albedo control in the off position. Again, report your results in graphical format. Explain the differences between the two sets of model runs. Can you relate your results to the concept of an ice-albedo feedback?

References


Acknowledgement

This model is completely based on that described by Tom Huber.