Chapter 24: Electric Current

Current

- **Definition of current**
  
  A current is any motion of charge from one region to another.

- Suppose a group of charges move perpendicular to surface of area \( A \).

- The current is the rate that charge flows through this area:

\[
I = \frac{dQ}{dt}; \quad dQ = \text{amount of charge that flows during the time interval } dt
\]

Units: 1 A = 1 ampere = 1 C/s
Current

Microscopic view of current

- In a conductor charges (electrons) are *always* in motion.
- They move with *speeds* of about $10^6$ m/s.
- If no electric *field* is applied the net electron *velocity is zero*.
- Then there is no net charge *flow*.
- When electric *field* is applied to the conductor it causes the electrons to *drift* in the opposite direction to the field.
Current

- Microscopic view of current (cont’d)

Collisions in the conductor causes the electrons to reach a steady average drift velocity, $v_d$. 
Current

Microscopic view of current (cont’d)

- In time $\Delta t$ the electrons move a distance $\Delta x = v_d \Delta t$
- There are $n$ particles per unit volume that carry charge $q$
- The amount of charge that passes the area $A$ in time $\Delta t$ is
  \[ \Delta Q = q(nA v_d \Delta t) \]
- The current $I$ is defined by:
  \[ I \equiv \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = nq v_d A \]
- The current density $J$ is defined by:
  \[ J = \frac{I}{A} = nq v_d \]
  Current per unit area
  Units: $\text{A/m}^2$
  \[ \vec{J} = nq \vec{v}_d \]
  Vector current density
## Resistivity

### Ohm’s law

- The current density $J$ in a conductor depends on the electric field $E$ and on the properties of the material.
- This dependence is in general complex but for some material, especially metals, $J$ is proportional to $E$.

$$J = \frac{E}{\rho}$$

Ohm’s law

$\rho$ : resistivity, Units $\text{V/m}/(\text{A/m}^2) = \text{V} \cdot \text{m/A} = \Omega \text{m}$

Conductivity $\sigma = 1/\text{resistivity}$

$$J = \sigma E$$

<table>
<thead>
<tr>
<th>Substance</th>
<th>$\rho$ ((\Omega\text{m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>silver</td>
<td>$1.47 \times 10^{-8}$</td>
</tr>
<tr>
<td>copper</td>
<td>$1.72 \times 10^{-8}$</td>
</tr>
<tr>
<td>gold</td>
<td>$2.44 \times 10^{-8}$</td>
</tr>
<tr>
<td>steel</td>
<td>$20 \times 10^{-8}$</td>
</tr>
<tr>
<td>graphite</td>
<td>$3.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>silicon</td>
<td>2300</td>
</tr>
<tr>
<td>glass</td>
<td>$10^{10} - 10^{14}$</td>
</tr>
<tr>
<td>teflon</td>
<td>$&gt;10^{13}$</td>
</tr>
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</table>
Resistivity

Conductors, semiconductors and insulators

• Good electrical conductors such as metals are in general good heat conductors as well. In a metal the free electrons that carry charge in electrical conduction also provide the principal mechanism for heat conduction.

• Poor electrical conductors such as plastic materials are in general poor thermal conductors as well.

• Semiconductors have resistivity intermediate between those of metals and those of insulators.

• A material that obeys Ohm’s law reasonably well is called an ohmic conductor or a linear conductor.
Resistivity

Resistivity and temperature

- The resistivity of a metallic conductor nearly always increases with increasing temperature.

\[
\rho(T) = \rho_0 [1 + \alpha(T - T_0)]
\]

- Temperature coefficient of resistivity
- Reference temperature (often 0 °C)

<table>
<thead>
<tr>
<th>Material</th>
<th>(\alpha (\degree C)^{-1})</th>
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<th>(\alpha (\degree C)^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>aluminum</td>
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<td>iron</td>
<td>0.0050</td>
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<tr>
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<td>lead</td>
<td>0.0043</td>
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<tr>
<td>graphite</td>
<td>-0.0005</td>
<td>manganin</td>
<td>0.000000</td>
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<tr>
<td>copper</td>
<td>0.00393</td>
<td>silver</td>
<td>0.0038</td>
</tr>
</tbody>
</table>
Resistivity

Resistivity vs. temperature

- The resistivity of graphite decreases with the temperature, since at higher temperature more electrons become loose out of the atoms and more mobile.
- This behavior of graphite above is also true for semiconductors.
- Some materials, including several metallic alloys and oxides, has a property called superconductivity. Superconductivity is a phenomenon where the resistivity at first decreases smoothly as the temperature decreases, and then at a certain critical temperature $T_c$ the resistivity suddenly drops to zero.
Resistance

- Resistance
  - For a conductor with resistivity $\rho$, the current density $J$ at a point where the electric field is $E$:
    \[ \vec{E} = \rho \vec{J} \]
  - When Ohm’s law is obeyed, $\rho$ is constant and independent of the magnitude of the electric field.
  - Consider a wire with uniform cross-sectional area $A$ and length $L$, and let $V$ be the potential difference between the higher-potential and lower-potential ends of the conductor so that $V$ is positive.
    \[ I = JA, \quad V = EL \]
    \[ \vec{E} = \rho \vec{J} \]
    \[ R \equiv \frac{V}{I} \]
    \[ E = \frac{V}{L} = \rho \frac{I}{A} \rightarrow V = \rho \frac{L}{A} I \]
  - As the current flows through the potential difference, electric potential is lost; this energy is transferred to the ions of the conducting material during collisions.

1 V/A = 1 Ω
Resistance

- As the resistivity of a material varies with temperature, the resistance of a specific conductor also varies with temperature. For temperature ranges that are not too great, this variation is approximately a linear relation:

\[ R(T) = R_0[1 + \alpha(T - T_0)] \]

- A circuit device made to have a specific value of resistance is called a resistor.
Resistance

Example: Calculating resistance

- Consider a hollow cylinder of length $L$ and inner and outer radii $a$ and $b$, made of a material with $\rho$. The potential difference between the inner and outer surface is set up so that current flows radially through cylinder.

- Now imagine a cylindrical shell of radius $r$, length $L$, and thickness $dr$.

$$A = 2\pi rL$$ : area of a cylinder represented by a dashed circle from which the current flows

$$dR = \frac{\rho dr}{2\pi rL}$$ : resistance of this shell

$$R = \int dR = \frac{\rho}{2\pi L} \int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L} \ln \frac{b}{a}$$
Electromotive Force (emf) and Circuit

Complete circuit and steady current

- For a conductor to have a steady current, it must be part of a path that forms a closed loop or complete circuit.

Electric field $\vec{E}_1$ produced inside conductor causes current

Current causes charge to build up at ends, producing opposing field $\vec{E}_2$ and reducing current

After a very short time $\vec{E}_2$ has the same magnitude as $\vec{E}_1$: total field $\vec{E}_{total} = 0$ and current stops completely.
Maintaining a steady current and electromotive force

- When a charge $q$ goes around a complete circuit and returns to its starting point, the potential energy must be the same as at the beginning.
- But the charge loses part of its potential energy due to resistance in a conductor.
- There needs to be something in the circuit that increases the potential energy.
- This something to increase the potential energy is called electromotive force (emf). Units: 1 V = 1 J/C
- Emf makes current flow from lower to higher potential. A device that produces emf is called a source of emf.

- If a positive charge $q$ is moved from $b$ to $a$ inside the source, the non-electrostatic force $F_n$ does a positive amount of work $W_n = qV_{ab}$ on the charge.
- This replacement is opposite to the electrostatic force $F_e$, so the potential energy associated with the charge increases by $qV_{ab}$. For an ideal source of emf $F_e = F_n$ in magnitude but opposite in direction.
- $W_n = q\varepsilon = qV_{ab}$, so $V_{ab} = \varepsilon = IR$ for an ideal source.
Electromotive Force (emf) and Circuit

- Internal resistance
  - Real sources in a circuit do not behave ideally; the potential difference across a real source in a circuit is not equal to the emf.

\[ V_{ab} = \varepsilon - Ir \] (terminal voltage, source with internal resistance \( r \))

- So it is only true that \( V_{ab} = E \) only when \( I = 0 \). Furthermore,

\[ \varepsilon - Ir = IR \text{ or } I = \frac{\varepsilon}{R + r} \]
Electromotive Force (emf) and Circuit

- Real battery

Real battery has internal resistance, $r$.

**Terminal voltage**, $\Delta V_{\text{output}} = (V_a - V_b) = \mathcal{E} - I \cdot r$.

\[
I = \frac{\Delta V_{\text{out}}}{R} = \frac{\mathcal{E} - I \cdot r}{R} \implies I = \frac{\mathcal{E}}{R + r}
\]
Electromotive Force (emf) and Circuit

Potential in an ideal resistor circuit
Electromotive Force (emf) and Circuit

Potential in a resistor circuit in realistic situation
Energy and Power in Electric Circuit

Electric power

Electrical circuit elements convert **electrical energy** into

1) **heat energy** (as in a resistor) or
2) **light** (as in a light emitting diode) or
3) **work** (as in an electric motor).

It is useful to know the **electrical power being supplied**.

Consider the following simple circuit.

\[ dU_e = dQ\Delta V = dQV_{ab} \]

\( dU_e \) is electrical **potential energy** lost as \( dQ \) traverses the resistor and falls in \( V \) by \( \Delta V \).

**Electric power** = rate of supply from \( U_e \).

Electric power \( P = \frac{dQ}{dt} V_{ab} = IV_{ab} = I^2 R = \frac{V_{ab}^2}{R} \)

Units: \((1 \text{ J/C})(1 \text{ C/s}) = 1 \text{ J/s} = 1 \text{ W} \text{ (watts)} \)
Energy and Power in Electric Circuit

- **Power output of a source**
  - Consider a source of emf with the internal resistance $r$, connected by ideal conductors to an external circuit.
  - The rate of the energy delivered to the external circuit is given by:
    $$ P = V_{ab} I $$
  - For a source described by an emf $\varepsilon$ and an internal resistance $r$
    $$ V_{ab} = \varepsilon - Ir $$
    $$ P = V_{ab} I = \varepsilon I - I^2 r $$

![Diagram showing a battery (source) and a headlight (external circuit)]
Energy and Power in Electric Circuit

Power input of a source

- Consider a source of emf with the internal resistance $r$, connected by ideal conductors to an external circuit.

\[ V_{ab} = \varepsilon + Ir \rightarrow P = V_{ab}I = \varepsilon I + I^2r \]

- Rate of conversion of electrical energy into nonelectrical energy in the battery.
- Rate of dissipation of energy in the internal resistance in the battery.

Total electrical power input to the battery.
Electromotive Force (emf) and Circuit

Examples:

\( r = 2 \, \Omega, \varepsilon = 12 \, \text{V}, R = 4 \, \Omega \)

\[
I = \frac{\varepsilon}{R + r} = \frac{12 \, \text{V}}{4 \, \Omega + 2 \, \Omega} = 2 \, \text{A}.
\]

\[
V_{ab} = V_{cd}.
\]

\[
V_{cd} = IR = (2 \, \text{A})(4 \, \Omega) = 8 \, \text{V}.
\]

\[
V_{ab} = \varepsilon - Ir = 12 \, \text{V} - (2 \, \text{A})(2 \, \Omega) = 8 \, \text{V}.
\]

The rate of energy conversion in the battery is \( \varepsilon I = (12 \, \text{V})(2 \, \text{A}) = 24 \, \text{W} \).

The rate of dissipation of energy in the battery is \( Ir^2 = (2 \, \text{A})^2 (2 \, \Omega) = 8 \, \text{W} \).

The electrical power output is \( \varepsilon I - I^2 r = 16 \, \text{W} \).

The power output is also given by \( V_{bc} I = (8 \, \text{V})(2 \, \text{A}) = 16 \, \text{W} \).

It is also given by \( IR^2 = (2 \, \text{A})^2 (4 \, \Omega) = 16 \, \text{W} \).
Electric Conduction

- Drude’s model

- In a conductor charges (electrons) are *always* in motion.
- Speeds of about $10^6$ m/s.
- If no electric field is applied the net electron *velocity is zero*.
- Then there is no net charge *flow*.
- When electric *field* is *applied* to the conductor it causes the electrons to *drift* in the opposite direction of the field.

\[ \vec{F} = -e\vec{E} \]
Electric Conduction

Drude’s model (cont’d)

\[ \vec{F} = -e\vec{E} = m_e \ddot{a} \implies \ddot{a} = \frac{-e\vec{E}}{m_e} \]

acceleration

Recall that \( \vec{v}_f = \vec{v}_i + \ddot{a}t \) so \( \vec{v}_f = \vec{v}_i + \frac{-e\vec{E}}{m_e} t \)

- The electrons collide with the atoms (or lattice ions) in the conductor.
- Drude assumed that after each collision electrons had a randomly directed velocity.
- The added velocity between collisions is \((e\vec{E}/m_e)t\).
- Let the average time between collisions be \( \tau \), average velocity is then

\[ \vec{v}_f = \vec{v}_i + \frac{-e\vec{E}}{m_e} \tau = \frac{-e\vec{E}}{m_e} \tau = \vec{v}_d \]

\( \vec{v}_d \) is known as the drift velocity of the electron.
Electric Conduction

Drude’s model (cont’d)

\[ \vec{v}_d = \frac{-e\vec{E}}{m_e\tau} \]

Last lecture we obtained current density in a conductor: \( \mathbf{J} = -ne\vec{v}_d \).
So we obtain here:

\[ \mathbf{J} = -ne\vec{v}_d = \frac{ne^2\vec{E}}{m_e\tau} \]

From Ohm’s Law \( \mathbf{J} = \sigma\mathbf{E} \) we have:

\[ \sigma = \frac{ne^2\tau}{m_e} \quad \text{and} \quad \rho = \frac{1}{\sigma} = \frac{m_e}{ne^2\tau} \]

Conductivity and resistivity of conductor
Exercise 1

Calculate the resistance of a coil of platinum wire with diameter 0.5 mm and length 20 m at 20° C given \( \rho = 11 \times 10^{-8} \, \Omega \cdot m \). Also determine the resistance at 1000° C, given that for platinum \( \alpha = 3.93 \times 10^{-3} \, /° C \).

\[
R_0 = \rho \frac{l}{A} = (11 \times 10^{-8} \, \Omega \cdot m) \frac{20 \, m}{\pi [0.5(0.5 \times 10^{-3} \, m)]^2} = 11 \Omega
\]

To find the resistance at 1000° C:

\[
\rho = \rho_0 \left[ 1 + \alpha \left( T - T_0 \right) \right]
\]

But \( R = \rho \frac{l}{A} \), so we have:

\[
R = R_0 \left[ 1 + \alpha \left( T - T_0 \right) \right]
\]

Where we have assumed \( l \) and \( A \) are independent of temperature - could cause an error of about 1% in the resistance change.

\[
R(1000°C) = (11 \Omega)[1+(3.93 \times 10^{-3} \, ^{°}C^{-1})(1000 \, ^{°}C-20 \, ^{°}C)] = 53 \Omega
\]
Exercise 2

A 1000 W hair dryer manufactured in the USA operates on a 120 V source. Determine the resistance of the hair dryer, and the current it draws.

\[
I = \frac{P}{\Delta V} = \frac{1000 \text{ W}}{120 \text{ V}} = 8.33 \text{ A}
\]

\[
\Delta V = IR \Rightarrow R = \frac{\Delta V}{I} = \frac{120 \text{ V}}{8.33 \text{ A}} = 14.4 \Omega
\]

The hair dryer is taken to the UK where it is turned on with a 240 V source. What happens?

\[
P = \frac{(\Delta V)^2}{R} = \frac{(240 \text{ V})^2}{14.4 \Omega} = 4000 \text{ W}
\]

This is four times the hair dryer’s power rating – BANG and SMOKE!