Measuring the Gravitational Acceleration with a Pendulum

In this lab you will measure the gravitational acceleration using a simple pendulum. For the purposes of this lab, the simple pendulum is modeled as a simple harmonic oscillator, but with the caveat that this only applies under small oscillations. This model predicts that the period the pendulum is independent of the amplitude of oscillation, and that the period is 

\[ T = 2\pi \sqrt{\frac{L}{g}} \]

where \( L \) is the length of the string, \( g \) is the gravitational acceleration, and \( T \) is the period.

Equipment

A simple pendulum, a meter stick, a protractor, a stopwatch or other timer (0.1 second or better precision).

Procedure

This lab has three parts. The first two parts verify the accuracy of the model for the simple pendulum. The final part measures the gravitational acceleration.

Verify the relationship between period and amplitude.

The “simple harmonic oscillator” model of simple pendulum predicts that for small oscillations, the period is independent of the amplitude. The first step in this lab is to investigate the range of amplitudes over which the small angle approximation is valid. This can be done by measuring the period for a variety of amplitudes and determining the range over which the amplitude does not change.

1. Adjust the length of the string so that it is about 50 cm. long.
2. Using the stop watch, measure the period for several amplitudes (e.g. 5°, 10°, 15°, 30°, 60°)
   2.1 Use the protractor to start the oscillation at a particular amplitude.
   2.2 Determine the period by measuring the time for the pendulum to go through at least ten cycles.
3. Plot the period as a function of the amplitude.
   3.1 Does the period vary as a function of amplitude? How much?
   3.2 Over what range of angle is the small angle approximation valid?

Verify the relationship between length and period.

The simple pendulum model predicts that the period squared, \( T^2 \), is linearly proportional to the length of the string, so the second step in this lab is to verify this prediction. This can be done by measuring the period (for a small amplitude) for various string lengths and checking that the predicted relationship is correct.

1. Using the stop watch, measure the period for several different string lengths (e.g. about 20 cm, 30 cm, 40 cm, 50 cm, and 60 cm). Be sure that you measure the length of the string from the
top attachment point to the **center** of the mass.

1. Use your results from the previous step to choose an appropriate amplitude.

1.2 Determine the period by measuring the time for the pendulum to go through at least ten cycles.

2 Plot the period squared, $T^2$, as a function of the string length, $L$.

2.1 Is the square of the period linearly proportional to the length of the string?

**Measure the acceleration due to gravity.**

The final step is to determine the acceleration due to gravity. Before doing that, you should consider how to get the most accurate measurement. There are three factors you must consider:

1 How accurately can you measure the period?

   1.1 To get an accurate measurement of the period, you should measure the pendulum for at least 100 seconds and 50 cycles (which ever takes longer).

   1.2 What is the uncertainty on the measured time ($\sigma_T$)?

      1.2.1 The fractional uncertainty is defined as $\frac{\sigma_T}{T}$. What is the fractional uncertainty?

2 How accurately can you measure the length of the string?

   2.1 Choose an appropriate length for the string. You should choose a length that is long enough to measure accurately.

   2.2 Measure how long you have made the string.

   2.3 What is the uncertainty on the measured length of the string ($\sigma_L$)?

      2.3.1 The fractional uncertainty is defined as $\frac{\sigma_L}{L}$. What is the fractional uncertainty?

3 How valid is the small angle approximation used in the model of the simple pendulum?

   3.1 Choose an appropriate starting amplitude.

4 Measure the period of the pendulum.

The acceleration due to gravity is then found using $g = 4\pi^2 \frac{L}{T^2}$ and the estimated fractional uncertainty on the gravitational acceleration is equal to $\frac{\sigma_g}{g} = \sqrt{\left(\frac{\sigma_L}{L}\right)^2 + 4\left(\frac{\sigma_T}{T}\right)^2}$ so you can determine the estimated uncertainty of your measurement.

1. What is your measured value for “$g$”? 

2. What is the fractional uncertainty for your measurement of “$g$”? 

3. How could you have improved your measurement?