Estimating the Precision on Future $\sin^2(2\theta_{13})$ Measurements

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CENTRAL QUESTION; What is the Probability (usually assumed 90%) for a given integ. lum. (usually in Kilowatt-year) that T2K will measure $\sin^2(2\theta_{13})$ to a certain precision.

• In this presentation the simple analytically techniques are shown how to estimate how precisely we can measure $\sin^2(2\theta_{13})$ in T2K.

• I will use results from Josh Albert’s analysis note and assume his estimates on the T2K detection efficiency and most importantly his estimate of the T2K background rates.

• I will use simple probability techniques and will not discuss the differences between classical, bayesian, and Feldman-Cousins probabilities.

• First lets see why these predictions are useful. They usually are made for comparisons between experiments in agency reviews of expts.
Propaganda slide made in a NOVA talk. An accurate estimate for T2K (solid red line), 0.015 by 2016???

Progress toward $\theta_{13} > 0$

90 and 99% CL measurements of $\sin^22\theta_{13}$

$\sin^22\theta_{13} = 0$, normal hierarchy, averaged over $\delta_{CP}$

Once Daya Bay reports (green curve), it dominates the combined sensitivity of all experiments (black curve) in the search for $\theta_{13} > 0$
INGREDIENTs needed to estimate the sensitivity
(from Josh Albert’s analysis note)

1. For a given integrated beam exposure, Kilowatt year (=10^7 sec), how many signal events for a given value of \( \sin^2(2\theta_{13}) \), after analysis cuts are expected
2. For the same period and same analysis cuts, how many background events are expected

⇒ For 750KW@5 years, after analysis cuts, there are expected 143 signal events and 25 background events. This assumes \( \sin^2(2\theta_{13}) = 0.1 \) & average \( \delta_{cp} \)

We can scale this down for early T2K running

⇒ For 100KW@1 year, after analysis cuts, there are expected 3.8 signal events and 0.67 background events. This assumes \( \sin^2(2\theta_{13}) = 0.1 \)

Note if \( \sin^2(2\theta_{13}) = 0.05 \), we get \( \frac{1}{2} \) as many events
Posing the key statistics question in an Equation

Lets assume now we know there will be $s$ signal events and $b$ background events. What is the gaussian probability (valid at large statistics) of observing $n$ or more events that are the sum of signal plus background)?

$$P(\text{obs} > n) = N \int_{\infty}^{n} e^{-\frac{(s+b-x)^2}{2\sigma^2}} dx$$

The 90% chance of observing any signal is translated into observing any number of events above the background $b$. This is defined by the following equation,

$$0.9 = P(\text{obs} > b) = N \int_{b}^{\infty} e^{-\frac{(s+b-x)^2}{2\sigma^2}} dx$$

So all that remains is to find $s$, so we can determine the minimum signal value $s$ that permits this 90% chance to occur.
Solving for s in the integral equation

- Lets assume the five year run at 750KW and that we will have a background known exactly to be, \( b = 25 \)
- the \( \sigma \) error is the square root of the #events, so \( \sigma = \sqrt{s+b} \)
- looking up in a definite integral table for \( P=0.9 \), finds,

\[
0.9 = N \int_{x_0 - 1.23\sigma}^{\infty} e^{-\left(\frac{(x_0-x)^2}{2\sigma^2}\right)} \, dx
\]

- for our integral we have,

\[
0.9 = N \int_{s+b - 1.23\sigma}^{\infty} e^{-\left(\frac{(s+b-x)^2}{2\sigma^2}\right)} \, dx
\]

and using the previous page’s integral lower limit equal to \( b \), this becomes,

\[
b = s + b - 1.23\sigma
\]

\[
s = 1.23\sigma = 1.23\sqrt{s+b}
\]
We can readily solve this quadratic equation for $s$ and obtain,

$$s = \frac{1 + \sqrt{1 + 4b/1.28^2}}{2/1.28^2}$$

And for $b=25$, we find,

$$s = \frac{1 + \sqrt{1 + 4(25)/1.28^2}}{2/1.28^2} = 7.27$$

SO, we have a 90% chance of seeing events above the Background of 25, if the true # signal events is 7.27. This corresponds to a value of $\sin^2(2\theta_{13}) = 0.1(7.27/143) = 0.005$

**RECAPPING**

In a 750KW 5 year run, we will have a 90% probability to see a signal above background, if the true signal is $\geq 7.27$ events which corresponds to a value of $\sin^2(2\theta_{13}) \geq 0.005$

We use the term 90% “confidence level” meaning that we are confident at 90% probability to observe a signal.
We can use our formula to plot the value of $\sin$ as a function of each 750KW year. It is straightforward to change our eqn as a function of 750KW years.

$$\sin^2(2\theta_{13}) = \left( \frac{0.1}{143} \right) \left( \frac{5}{x} \right) \left( \frac{1 + \sqrt{1 + 4(5x)/1.28^2}}{2 / 1.28^2} \right)$$

We find the precision rapidly improves and that in 2 years, we should reach 0.01 precision on $\sin^2(2\theta_{13})$. 

![Graph showing the decrease in $\sin^2(2\theta_{13})$ with increasing 750KW years.]
Estimates for 100 KW year run

For the beginning run this fall, we assume a 100 KW run for one year ($10^7$ sec). In this case we predicted having 0.67 background events and 3.8 signal events if $\sin^2(2\theta_{13}) = 0.1$. Since these are small statistics, we should use Poisson statistics.

Recall that Poisson statistics are given by,

$$P(n) = \frac{\lambda^n e^{-\lambda}}{n!}$$

where $\lambda = s + b$ and $P(n)$ is the probability to observe the $n$ events. Following the previous recipe, we require for 90% probability where the initial value $n_i > b$.

Since $b = 0.67$, we must have $n_i = 1$ and need now to solve for $s$ for given value for $b$. 

$$0.9 = \frac{\sum_{n=0}^{\infty} P(n)}{\sum_{n=n_i}^{\infty} P(n)}$$
Continuing, we need to solve the following for $s$,

$$0.9 = \frac{\sum_{n=1}^{\infty} P(n)}{\sum_{n=0}^{\infty} P(n)} = \frac{\sum_{n=0}^{\infty} P(n) - P(0)}{\sum_{n=0}^{\infty} P(n)} = \frac{\sum_{n=0}^{\infty} P(n) - e^{-\lambda}}{\sum_{n=0}^{\infty} P(n)}$$

Using the fact that, $1 = \sum_{n=0}^{\infty} P(n)$

We have that, $0.9 = 1 - e^{-\lambda} = 1 - e^{-(s+b)}$

And we readily find,

$$s = \ln(10) - b = \ln(10) - 0.67 = 2.3026 - 0.67 = 1.63$$

So we predict 1.63 signal events as the 90% Poisson probability. This corresponds to $\sin^2(2\theta_{13})=0.1(1.63/3.8)=0.043$
RECAPPING the Poisson distribution prediction

If we operate T2K for one year with average 100KW beam, we have a 90% probability to observe a 1 event, which is above the predicted background of 0.67, if the true value of the signal is 1.63 events. This corresponds to \( \sin^2(2\theta_{13}) = 0.043 \) at a 90% confidence level.

ALTERNATE approach to predicting the Sensitivity;
Another approach is to ask what is the value of \( \sin^2(2\theta_{13}) \) that will create a signal that is 3 standard deviations above the background. A 3 standard deviation effect for Gaussian probability corresponds to 97.3%. An equivalent interpretation is an upward background fluctuation has a 2.7% probability that corresponds to a signal above background at 3\( \sigma \). We can formulate the problem as given that the background is \( b=0.67 \), how many events \( n' \) must we observe such the probability is 2.7%. Then the value of the 3\( \sigma \) signal = \( n'-b \)
We can formulate the probability as,

\[ 0.027 = \sum_{n=n'}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} \]

Where, \( \lambda = b = 0.67 \). We need to find \( n' \), such that the above is satisfied. By trial and error, we find

\[ \sum_{n=3}^{\infty} \frac{0.67^n e^{-0.67}}{n!} = 0.0306 \]

If we observe 3 events, then we have an upward fluctuation of 3 - 0.67 = 2.33 events. This corresponds to \( \sin^2(2\theta_{13}) = 0.1 \times (2.33/3.8) = 0.061 \).

So a 3σ signal will be seen in a 100KW one year run if we have \( \sin^2(2\theta_{13}) = 0.061 \).
FINAL COMMENTS

1) If the signal is small, then the accuracy of the background prediction is very important. In our estimates we have neglected the uncertainty in the background. If we believe the background uncertainty can be represented by a Gaussian, we can convolute or smear the equations by varying the background and weighting the results with a Gaussian. The measurement of the neutral current $\pi^0$ background in H$_2$O will be very important since it is an irreducible background and a direct subtraction of our candidate events. It is by itself equal to a “pedestal” $\sin^2(2\theta_{13})=0.1(25/143)=0.017$.

2) If the systematic error is large, then all bets are off since this introduces subjective interpretation. An example is if the particle distributions of Monte Carlo backgrounds do not match what is observed in the detector.