Problem 12.27

Description: The figure shows a traffic signal, with the masses and positions of its various members indicated. The structure is mounted with two bolts, located symmetrically about the vertical member's centerline, as indicated. What tension force must the left-hand bolt be capable of withstanding?

Solution

The forces on the traffic signal structure, and their lever arms about point O (on the vertical member’s centerline between the bolts) are shown above. The normal force exerted by the bolts and the ground on the vertical member are designated by $n_l$ and $n_r$, measured positive upward. The two conditions of static equilibrium needed to determine $n_l$ and $n_r$ are:

$$0 = (\sum F_y)_o = n_l + n_r - (9.8 \text{ m/s}^2)(320 \text{ kg} + 170 \text{ kg} + 65 \text{ kg})$$

and

$$0 = (\sum \tau_z)_o = (n_r - n_l)(0.38 \text{ m}) - [(170 \text{ kg})(3.5 \text{ m}) + (65 \text{ kg})(8.0 \text{ m})](9.8 \text{ m/s}^2)$$

These two equations can be rewritten as:

$$n_r + n_l = 5.44 \times 10^3 \text{ N}, \quad n_r - n_l = 2.88 \times 10^4 \text{ N}$$

Thus we find $n_l = -1.17 \times 10^4 \text{ N}$, which is downward and must be exerted by the bolt.
Description: A garden cart loaded with firewood is being pushed horizontally when it encounters a step 8.0 cm high, as shown in the figure. The mass of the cart and its load is 55 kg, and the cart is balanced so that its center of mass is directly over the axle. The wheel diameter is 60 cm.

Part A
What is the minimum horizontal force that will get the cart up the step?
Express your answer using two significant figures.

Solution
We assume that a horizontal push on the cart results in a horizontal force exerted on the wheels by the axle, as shown. We also assume that both wheels share the forces equally so that they can be treated together. Also shown in the figure are the weight of the cart and the normal force of the ground, both acting through the center of the wheels, and the force of the step $F_s$. If we consider the sum of the torques about the step, the latter does not contribute, and the wheels (and cart) will remain stationary as long as

$$\sum r_{\text{step}} = MgR \sin \theta - nR \sin \theta - FR \cos \theta = 0.$$ 

When $n = 0$, however, the wheels begin to lose contact with the ground and go over the step, so

$$\theta = \cos^{-1}(1 - h/R) = \cos^{-1}(1 - 8/30).$$

Then

$$F = (55 \times 9.8 \text{ N}) \tan(\cos^{-1}(11/15)) = 500 \text{ N}$$ is the minimum force.
Problem 12.37

**Description:** The figure shows a 66 kg sign hung centered from a uniform rod of mass 8.2 kg and length 2.3 m. At one end the rod is attached to the wall by a pivot; at the other end it's supported by a cable that can withstand a maximum tension of 800 N.

Part A

What is the minimum height above the pivot for anchoring the cable to the wall? **Express your answer using two significant figures.**

**Solution**

From the equilibrium condition, \( \sum \tau = TL \sin \theta - Mg(L/2) = 0 \), where \( T \) is the tension on the cable, \( L \) is the length of the rod and \( M \) is total mass of the sign and the rod. Solving for the tension

\[
T = Mg/(2 \sin \theta) = (1/2)Mg \sqrt{1 + L^2/h^2},
\]

where we have used \( \tan \theta = h/L \) and the identity \( 1 + \cot^2 \theta = 1/\sin^2 \theta \). Solving for \( h \), we obtain

\[
h = \left[ \frac{2T/(Mg)}{1} - 1 \right]^{-1/2}.
\]

Given the maximum tension that could be withstood in the cable, \( T_{\text{max}} \), the condition that must be met by \( h \) for the sign to remain in static equilibrium is

\[
h \geq \left[ \frac{2(T_{\text{max}}/Mg)}{1} - 1 \right]^{-1/2} = (2.3 \text{ m}) \left[ \frac{2(800 \text{ N})}{(66 \text{ kg} + 8.2 \text{ kg})(9.8 \text{ m/s}^2)} \right]^{-1} = 1.17 \text{ m}.
\]

So the minimum height \( h_{\text{min}} = 1.17 \text{ m} \).
Problem 12.57

Description: A uniform pole of mass $M$ is at rest on an incline of angle $\theta$, secured by a horizontal rope as shown in the figure.

Part A

What is the minimum coefficient of friction that will keep the pole from slipping?

Solution

From the equilibrium conditions:

\[
0 = \sum F_\parallel = f + T \cos \theta - Mg \sin \theta
\]

\[
0 = \sum F_\perp = n - T \sin \theta - mg \cos \theta
\]

\[
0 = \sum \tau = T(L/2) \cos \theta - f(L/2).
\]

The solutions for the force are $f = (1/2)Mg \sin \theta$, $T = (1/2)Mg \tan \theta$, and

\[
n = \frac{1}{2} Mg \left( 2 \cos \theta + \frac{\sin^2 \theta}{\cos \theta} \right)
\]

subject to the condition that $f \leq \mu n$. Therefore

\[
\sin \theta \leq \mu \left( 2 \cos \theta + \frac{\sin^2 \theta}{\cos \theta} \right) \quad \Rightarrow \quad \mu \geq \frac{\tan \theta}{2 + \tan^2 \theta}.
\]
**Problem 12.61**

**Description:** The figure shows a wheel on a slope with inclination angle 20°, where the coefficient of friction is adequate to prevent the wheel from slipping; however, it might still roll. The wheel is a uniform disk of mass 1.50 kg, and it is weighted at one point on the rim with an additional 0.950 kg mass.

**Part A**

Find the angle θ shown in the figure such that the wheel will be in static equilibrium. **Express your answer using two significant figures.**

**Solution**

The cw torque by the wheel about the contact point is \( \tau_1 = MgR \sin \theta \). The ccw torque by the weight is \( \tau_2 = mgd = mgR(\cos \phi - \sin \theta) \). From the equilibrium condition \( \tau_1 = \tau_2 \).

So \( (M + m) \sin \theta = m \cos \phi \rightarrow \phi = \cos^{-1}\left[\frac{M + m}{m} \sin \theta \right] \).